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Stability Analysis of Randomly Perturbed Power Systems Through Index-Based Approaches

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1. Introduction

Stability at operating points is one of the key requirements of engineering systems. As long as the system is given by time-invariant dynamics, linearization at the operating point gives local stability information that can be extended through incorporating some nonlinear features, e.g., via the use of normal forms, see [1]. If the system under consideration has timevarying dynamics, the usual modal approach fails since for these systems eigenvalues do not describe the stability behavior of the linearized system. Therefore, one has to approach (exponential) stability directly via the Lyapunov exponents of the system at the operating point. An important class of systems with time varying dynamics are those systems that are subject to sustained random perturbations, such as load behavior, environmental effects, or intermittent generation in power systems. The interaction between system dynamics and perturbation falls into two groups: (i) the random noise changes the operating point of the system, or (ii) the equilibrium point persists under all perturbations. We have developed performance indices for case (i) in [2], and analyzed one specific approach in case (ii) in [3] using almost sure Lyapunov exponents. This paper develops several performance indices for case (ii), analyzes their relationships, and compares the results for several examples. The key idea is the look at the sample (exponential) growth rates for trajectories and at the growth rates of moments of the trajectories, such as the stability of the mean, or mean square stability involving the second moment. Both points of view result in potentially useful performance criteria for power systems.

Mathematical background

The system model

We start from a nonlinear differential equation y(t)=f(y(t),ε(t,ω)) in R^d with sustained random perturbation ε (t,ω). In order to analyse optimal parameter settings for stability at an operating point, we linearize the system equations at the equilibrium point y. Linearization (with respect to y) at the equilibrium results in the system where A(ε (t,ω)); is the Jacobian of *f*(y,(t), ε (t,ω)at y. We denote ε (y,ω) by the trajectories of (1) with initial value uð0; x;xÞ ¼ x 2 Rd . We think of a given probability space ðX; F; PÞ under the usual conditions on which the Wiener process in (2) is defined. We use the notation x 2 X, and all expectations EðÞ are with respect to the given probability measure P. The random perturbation can be considered as white noise, leading to a stochastic differential equation for (1), or as a colored, bounded noise. In this paper we discuss the latter situation since macroscopic perturbations in engineering systems generally are non-white; but a similar theory also holds for the white noise case, see [4] for the basics. We start from a background noise g, given by a stochastic differential equation on a compact C1-manifold M

$$d\eta = X_0(\eta)dt + \sum_{i=1}^{r} X_i(\eta) \circ dW_i \text{ on } M$$

where the vector fields $X_0,\dots,X_r$ are C^∞, and "$\circ$" denotes the Stratonovic stochastic differential. We assume that (2) has a unique stationary, ergodic solution which is guaranteed by the condition (compare [5])

$$\dim \mathcal{LA}\{X_1,\dots,X_r\}(\theta) = \dim M \text{ for all } \theta \in M.$$

Here LAfg denotes the Lie algebra generated by a set of vector fields. The background noise gðt;xÞ is mapped via a surjective smooth function f :

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$\eta^*(t,\omega)$, into the system perturbation nðt;xÞ. This setup allows great flexibility when modeling the statistics of the system noise.

$$\lambda(x,\omega) = \limsup_{t\to\infty} \frac{1}{t} \log|\varphi(t,x,\omega)|,$$

and for $p \in \mathbb{R}$ the Lyapunov exponent of the pth moment is given by

$$g(p,x) = \limsup_{t\to\infty} \frac{1}{t} \log \mathbb{E}|\varphi(t,x,\omega)|^p.$$

This includes for p =1 the exponential growth behavior of the mean, and for p =2 the exponential mean square stability of the system. We again need the projection of the linear system onto the sphere S^{d-1} in R^d:

$$\dot{s}(t) = h(\xi(t,\omega), s(t)),\ h(\xi,s) = (A(\xi) - q(\xi,s))s,\ q(\xi,s) = s^T A(\xi)s,$$

where ‘‘ T ’’ denotes the transpose. via identification of s and -s Eq. (6) can be considered on the projective space P^{d-1}. The Lyapunov exponents of all system states x ϵ $R^d \setminus \{0\}\}$ can be analyzed together if the perturbation affects all states. This is expressed in the condition

$$\dim \mathcal{LA}\left\{\left(X_0, h, \frac{\partial}{\partial t}\right), (X_1,0,0),\ldots,(X_r,0,0)\right\}(\theta,s,t) = \dim M + d$$

for all $(\theta,s,t) \in M \times \mathbb{S}^{d-1} \times \mathbb{R}$. Another approach to condition (7) is as follows: Let I be the ideal in $\mathcal{LA}\{X_0 + h, X_1,\ldots,X_r\}$ generated by $\{X_1,\ldots,X_r\}$. Then, by [5], Condition (7) is equivalent to dimI I(O,S)h= dimM +d = 1. This condition, which is needed for the analysis of moment Lyapunov exponents, is slightly stronger than Condition 7 in [6], but it is generally satisfied for systems that appear in applications, compare, e.g., [5] or [10].

Theorem 2.1. Consider the stochastic system (1) under the conditions (3) and (7). Then

1. the moment Lyapunov exponents exist as a limit and they are independent of $x \in \mathbb{R}^d \setminus \{0\}$, i.e., $g(p) \equiv g(p,x) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}|\varphi(t,x,\omega)|^p$ for all $p \in \mathbb{R}$,

2. the trajectory-wise Lyapunov exponents are a.s. constant and independent of $x \in \mathbb{R}^d \setminus \{0\}$, i.e., $\lambda \equiv \lambda(x,\omega) = \lim_{t\to\infty} \frac{1}{t} \log|\varphi(t,x,\omega)|$.

The proof of Theorem 2.1 is given in [7], Theorem 1 for the first part, and in [10], Theorem 4.1 for the second part upon noticing that Conditions (3) and (7) together imply Conditions (A) and (C) in [10]. With the results from Theorem 2.1 it was shown by Arnold in [7] that the a.s. Lyapunov exponent is the derivative of the moment Lyapunov exponent function at 0:

Corollary 2.2. Consider the stochastic system (1) under the conditions (3) and (7). Then the function g(p) is analytic on R, convex, and satisfies g(0) = 0 and g'(0) =ʎ.

Remark 2.3. The information contained in Corollary 2.2 shows that

1. If the a.s. Lyapunov exponent of the system (1) is negative, i.e., if the system is almost surely exponentially stable, then moments for small p > 0 are also exponentially stable. And vice versa, if moments for small p > 0 are exponentially stable, then the system is a.s. exponentially stable.

2. The moment exponent function g(p) has at most two zeros. Specifically, if the system is a.s. exponentially stable, i.e., if k < 0, then g(p) has at most one zero besides g(0) = 0, and this occurs for p > 0. Assume that such a second zero exists at a > 0, then all pth moments are stable for 0 < p < a, and unstable for a < p < ꝏ.

In light of Remark 2.3 the key question regarding the exponential stability of the moments of (1) is the existence of a second zero of g(p), i.e., the existence of a > 0 with g(a) = 0. Such a point a does not always exist since g(a) = ʎp is possible, compare [9], Theorem 2.3, Case 2.1.2(a). A detailed analysis of this question involves the function c : R -R, given by

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$$\gamma(0) = \lambda, \gamma(p) = \frac{g(p)}{p} \quad \text{for } p \neq 0.$$

According to Corollary 2.2 this function is analytic on R and increasing. We define

$$\gamma^+ = \lim_{p\to\infty} \gamma(p)$$

and note that either $\gamma^+ = \lambda$, in which case $g(p) = \lambda p$, or $\lambda = \gamma(0)$ exists $a > 0$ with $g(a) = 0$ if and only iff $\gamma^+ > 0$. For the follow

Stability-based performance indices for stochastic systems

Performance indices

In this section we discuss performance indices for systems under stochastic perturbations. The goal is to identify system parameters that allow for optimal (exponential) stability behavior of a system at a stable operating point. Since exponential stability can be inferred from the system linearization, we consider systems as in (1), and we assume that the perturbation can be modeled by a function of a Markov-diffusion process as in (2). More specifically, our goal is to guarantee stability of the system under the largest possible perturbation range.

The size of the random perturbation is described in the following way: We consider the noise range U c R^m to be convex, compact with 0 ϵ int U, the interior of U. Introducing the size parameter P≥0 we consider Uq := q U together with the maps f q : M - U^p, f^p (Θ)=p.f(Θ).In this way we obtain a family ɛ̧P(t,ω) ,p≥0 of system perturbations with corresponding dynamics (1)q. For q = 0 this model corresponds to the unperturbed system. To be precise, we analyse the family of systems

$$\dot{x}^\rho(t) = A(\xi^\rho(t,\omega), b)x^\rho(t), \quad x \in \mathbb{R}^d, \quad \xi^\rho(t,\omega) \in U^\rho \subset \mathbb{R}^m,$$

where b ϵ Bc R^k is a vector of parameters that are to be tuned in such a way that the system (10) is stable for p ≥0 as large as possible.

The almost sure stability radius

$$r = \inf\{\rho \geqslant 0, \lambda(\rho) > 0\}$$

was introduced in [12] and analyzed in detail in [3]. Here kðqÞ denotes the a.s. Lyapunov exponent of (10). In a similar way, one can define the pth moment stability radius as

$$r(p) = \inf\{\rho \geqslant 0, g^\rho(p) > 0\} \text{ for each } p \in \mathbb{R}$$

with gÞ (p) as defined in (5) for the system (10). This stability radius provides an appropriate performance index if emphasis is placed on stability of specific moments of a system, such as the mean (p = 1) or mean square stability (p = 2). In both cases the design problem can be written as

$$\max_{b\in B} a(\rho, b) \quad \text{for a given } \rho > 0.$$

Section 2.2 points at other performance indices that can be useful for the evaluation of stability: Specifically, the second zero a(Þ) > 0 of the moment Lyapunov exponent function gÞ(p) of (10) not only describes the moments that are exponentially stable (see Remark 2.3), but also the boundedness behavior of individual trajectories (see Remark 2.5). For a given stochastic perturbation with given range Uq the design problem reads in this case

$$\max_{b\in B} a(\rho, b) \quad \text{for a given } \rho > 0.$$

This radius turns out to be described by the maximal Bohl exponent, or the maximum of the Morse spectrum of a deterministic perturbation (or control) system associated with (10), see [13], Chapter 7 for a detailed discussion of these concepts. In the following sections, we will analyze the moment stability radius and the stability index problem together with their relationships to the a.s. stability radius.

Computation of indices

While the computation of a.s. Lyapunov exponents has attracted great interest (see, e.g., [6,11,16], or [17] and the references therein), the computation of moment Lyapunov exponents seems relatively unexplored. One way is to write gÞ(p) as the maximal eigenvalue of a certain second order partial differential operator (see, e.g., [7] for the white noise, and [9] for the general case). This idea has been followed, e.g., in [21], in some examples of Chapter 9 in [20], or in [18], but generalizations to high dimensional systems appear more than cumbersome.

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The other approach is to follow the definition (5), i.e., simulate trajectories of the system (1), compute the moments and their exponential growth rate, see e.g., [19] or [20], Chapter 9.2 for an idea in this direction. Our experiences from [6] suggest to simulate solutions directly from the linear differential equation, using renormalization at regular time intervals since the trajectories grow or decay exponentially. This leads to the following approach:

Computing the expectation on each time interval $[t-1,t]$ for $t=1,\dots,T$ and $j=1,\dots,\alpha$ we obtain

$$\mathbb{E}|\varphi(T,x_0^j,\omega)|^p = \frac{\mathbb{E}|\varphi(T,x_0^j,\omega)|^p}{\mathbb{E}|x_0^j|^p} = \prod_{t=1}^{T}\frac{\mathbb{E}|\varphi(t,x_0^j,\omega)|^p}{|\mathbb{E}|\varphi(t-1,x_0^j,\omega)|^p|} \approx \prod_{t=1}^{T}\frac{\mathbb{E}|x_t^j(i)|^p}{|\mathbb{E}|s_{t-1}^j(i)|^p|} \tag{14}$$

and hence

$$\frac{1}{T}\log \mathbb{E}|\varphi(T,x_0^j,\omega)|^p = \frac{1}{T}\sum_{t=1}^{T}\log\frac{\mathbb{E}|\varphi(t,x_0^j,\omega)|^p}{|\mathbb{E}|\varphi(t-1,x_0^j,\omega)|^p|} \approx \frac{1}{T}\sum_{t=1}^{T}\log \mathbb{E}|x_t^j(i)|^p. \tag{15}$$

We now note that $\mathbb{E}|x_t^j(i)|^p = \frac{1}{\beta}\sum_{i=1}^{\beta}|x_t^j(i)|^p$ by averaging over the realizations of the background process, and finally averaging over all α initial values we obtain

$$g(p,x) = \lim_{T\to\infty}\frac{1}{T}\log \mathbb{E}|\varphi(T,x,\omega)|^p \approx \frac{1}{\alpha}\frac{1}{T}\sum_{j=1}^{\alpha}\sum_{t=1}^{T}\log\left(\frac{1}{\beta}\sum_{i=1}^{\beta}|x_t^j(i)|^p\right). \tag{16}$$

The numerical approximation of moment Lyapunov exponents given by Formula (16) uses only numerical solutions of (1) on time intervals of length t ðt 1Þ ¼ 1 and hence avoids solutions growing or decreasing exponentially over a long period of time. If time intervals of length 1 are too long to give reliable numerics for specific systems, this approach can easily be adapted to smaller intervals. Of course, burn-in intervals and choice of initial values have to be considered carefully, see [6] for a discussion of these issues for a.s. Lyapunov exponents; these considerations apply as well to the computation of moment exponents.

The performance indices introduced in Section 3.1 depend on moment Lyapunov exponents, and they require optimization with respect to the size q of the perturbation range and/or with respect to the tunable parameters b 2 B of the system. Given the general setup we have provided in the previous sections, we do not expect analytical results on these optimization problems. Therefore, optimization is performed numerically over a grid in the parameter spaces.

Examples

Three-dimensional linear oscillator

Consider the linear oscillator $\dot{x} = A(\xi_t)x$ in dimension 3 given by

$$\dot{x} = A(\xi_t)x = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c(1+\xi(t)) & -b & -a \end{pmatrix} x \quad \text{with } x \in \mathbb{R}^3.$$

For the computations we have used the values $a = 1$, $b_{nom} = 2$, and $c = 1$. The stochastic perturbation is $\xi^{\rho}(t) = \rho \cdot \sin(\eta(t))$, where $\eta(t,\omega)$ is an Ornstein-Uhlenbeck process as in [6] and $\rho \geqslant 0$ is the size of perturbation.

For the following discussion we consider b to be the tunable parameter with values in $B = [0.8b_{nom}, 1.2b_{nom}]$, and the size of perturbation satisfies $\rho \in [0, 1.4]$.

Fig. 1 shows the moment Lyapunov exponent curves for $b = 0.8b_{nom}$ and $\rho \in [0, 1.4]$.

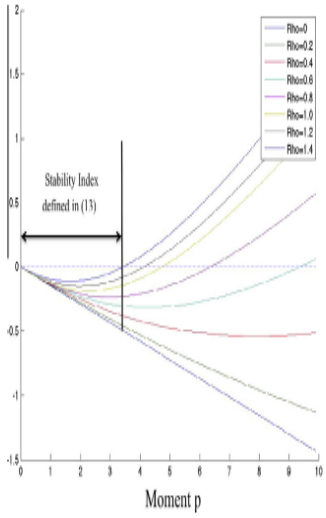

Fig. 1. Moment Lyapunov exponents of the linear oscillator, $b = 0.8b_{nom}$.

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Table 1
Almost sure Lyapunov exponents for $\rho = 0.8$.

$\rho = 0.8$	$b = 0.8b_{nom}$	$b = 0.9b_{nom}$	$b = 1.0b_{nom}$	$b = 1.1b_{nom}$	$b = 1.2b$
$\lambda(\rho)$	−0.15831	−0.19362	−0.22268	−0.25087	−0.270
$\gamma^{\rho}(0)$	−0.15758	−0.19363	−0.22349	−0.25225	−0.271

Table 2
Almost sure Lyapunov exponents for $\rho = 1.4$.

$\rho = 1.4$	$b = 0.8b_{nom}$	$b = 0.9b_{nom}$	$b = 1.0b_{nom}$	$b = 1.1b_{nom}$	$b = 1.2b$
$\lambda(\rho)$	−0.13345	−0.16359	−0.19494	−0.22503	−0.241
$\gamma^{\rho}(0)$	−0.13071	−0.16141	−0.19348	−0.22415	−0.240

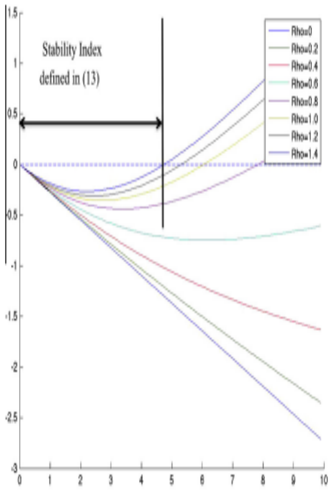

Fig. 2. Moment Lyapunov exponents of the linear oscillator, $b = 1.2b_{nom}$.

where v_1, v_2, v_3 are variables of the power system stabilizer, using expressions defined in [15]. The matrix A has the structure

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} \\ 0 & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} \end{pmatrix}.$$

The model for the field circuit is

$$\Delta\Psi_{fd} = \frac{K_3}{1 + sT_3}\left(\Delta E_{fd} - K_4 \Delta\delta\right)$$

and the excitation system is given by

$$\Delta E_{fd} = -K_A \Delta v_1,$$

where v_1 is the output of voltage transductor. The perturbation has been introduced as an error in the reference signal. This situation is described by changing the element a_{34} in the matrix A to

$$\Delta E_{fd} = -K_A(1 + \xi_t)\Delta v_1.$$

To be precise, we consider the (linearized) one machine - infinite bus system $\dot{x} = A(\xi_t)x$ with system matrix

$$A(\xi_t) = \begin{pmatrix} 0 & -0.11 & -0.12 & 0 & 0 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.19 & -0.42 & a_{34} & 0 & 27.4 \\ 0 & -7.3 & 20.8 & -50 & 0 & 0 \\ 0 & -1 & -1.1 & 0 & -0.71 & 0 \\ 0 & -4.8 & -5.4 & 0 & 26.9 & -30.3 \end{pmatrix},$$

where $a_{34} = a_{34} \cdot (1 + \xi_t)$ is a stochastic perturbation in the excitation component of the system. The stochastic perturbation is $\xi_t = \rho \cdot \sin(\eta_t)$ with η_t an Ornstein–Uhlenbeck process as in [6] and ρ is the size of perturbation.

In the similar case of Example 4.1, the key parameter in this system is the gain of the PSS, K_{PSS}, whose nominal value $K_{PSS_{nom}}$ was chosen as in [15]. In order to compute Performance Indices, we used gain values $K_w := w \cdot K_{PSS_{nom}}$, with $w \in [0.8, 1.2]$. For the range of the random perturbation we used $\rho \in [0, 0.5]$, with a step size of 0.1.

Fig. 3 shows the moment Lyapunov exponents curves for $K_w = 0.8 \cdot K_{PSS_{nom}}$ and $\rho \in [0, 0.5]$.

Fig. 3 allows us to determine the pth moment stability radius as in (12): we have, e.g., $r(2; K_w = 0.8 \cdot K_{PSS_{nom}}) = 0.1$, or $r(1.5; K_w = 0.8 \cdot K_{PSS_{nom}}) = 0.19$. Similar to the findings of Example 4.1, Fig. 3 shows the stability index proposed in (13) which corresponds to the moment p, for a fixed ρ, where system will not remain stable. In this case if the size of the perturbation is $\rho = 0.5$, then all moments $p \geqslant 1.2$ of the system will be unstable.

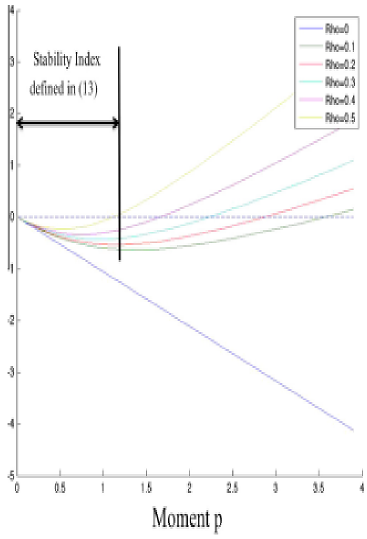

Fig. 3. Moment Lyapunov exponents of the one machine system, $K_w = 0.8K_{PSS_{nom}}$.

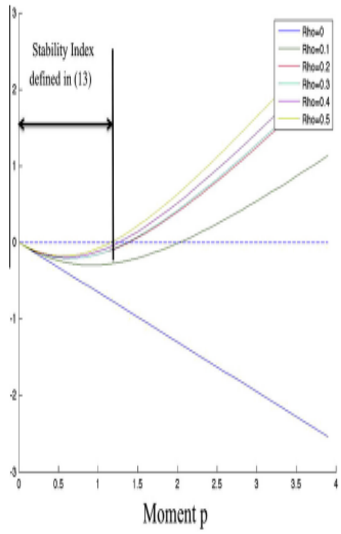

Fig. 4. Moment Lyapunov exponents of the one machine system, $K_w = 1.2K_{PSS_{nom}}$.

Fig. 4 shows the moment Lyapunov exponents curves for $K_w = 1.2 \cdot K_{PSS_{nom}}$ and $\rho \in [0; 0.5]$. Comparing the two cases $K_w = 0.8 \cdot K_{PSS_{nom}}$ in Fig. 3 and $K_w = 1.2 \cdot K_{PSS_{nom}}$ in Fig. 4 we see that the second zeros of the moment Lyapunov exponents curves are not monotone in K_w: we have $a(0.5, 0.8 \cdot K_{PSS_{nom}}) > a(0.5, 1.2 \cdot K_{PSS_{nom}})$, while $a(\rho, 0.8 \cdot K_{PSS_{nom}}) > a(\rho, 1.2 \cdot K_{PSS_{nom}})$ for all $\rho \in [0.1, 0.4]$. This indicates that optimal parameter tuning relative to exponential moment stability cannot simply be achieved by increasing PSS gains.

Conclusions

This paper proposes several performance indices for the stability of operating points in dynamical systems affected by sustained random perturbations. These indices are based on moment Lyapunov exponents and they complement the almost sure Lyapunov exponent and stability radius analyzed in [6,2]. The two sets of indices are related by the fact that the almost sure Lyapunov exponent of a system is the derivative at p = 0 of the pth moment Lyapunov exponent function g(p). This means, in particular, that for small moments p > 0 the pth moment Lyapunov exponents contain the 'same' information as the a.s. exponent, while for large moments p ≥≥ 0 the pth moment Lyapunov exponents contain the 'same' information as the maximal deterministic (robust) exponent $y^{þ}$, compare Remark 2.4. Hence for design purposes stability indices based on moment Lyapunov exponents can be used to strike a balance between almost sure behaviour based on specific random perturbations, and behaviour based on the range of the perturbation. Design issues surrounding moment stability indices are the topic of a forthcoming paper.

Note that while we always have for the moment Lyapunov function g(p) that g(p) = 0, for a stable operating point the second zero of g(p), i.e., the point a > 0 with g(p) =0 determines the moment stability behavior. In realistic systems, such as the one machine – infinite bus power system, this second zero a may not depend in a monotone fashion on the size of the random perturbation, and on the amount of damping in the system. This indicates that optimal parameter tuning relative to exponential moment stability cannot simply be achieved by increasing system damping, such as PSS gains.

A key question then is to what extend system design that uses indices based on almost sure or moment Lyapunov exponents, depends on the specific statistics of the system noise ε(t,ω). Of course, if one wants to immunize a system against all specific noise statistics, one will have to use the deterministic (robust) exponent y^{+}: this index depends only on the size of the perturbation, not on its statistics, see the comments above. The robustness of the design indices presented in this paper relative to noise statistics is the topic of on-going research.

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